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QCD Sum-Rule Calculation of the Kinetic Energy and Chromo-Interaction of Heavy Quarks Inside Mesons

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Abstract

We present a QCD sum-rule determination of the heavy-quark kinetic energy inside a heavy meson, $-\lambda_1/2m_Q$, which is consistent with the field-theory analog of the virial theorem. We obtain $-\lambda_1 \approx (0.10 \pm 0.05) \text{ GeV}^2$, significantly smaller than a previous sum-rule result, but in good agreement with recent determinations from the analysis of inclusive decays. We also present a new determination of the chromo-magnetic interaction, yielding $\lambda_2(m_b) = (0.15 \pm 0.03) \text{ GeV}^2$. This implies $m_{B^*}^2 - m_B^2 = (0.60 \pm 0.12) \text{ GeV}^2$, in good agreement with experiment. As a by-product of our analysis, we derive the QCD sum rules for the three form factors describing the meson matrix element of a velocity-changing current operator containing the gluon field-strength tensor.

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1 Introduction

The physics of hadrons containing a heavy quark simplifies greatly in the limit where the heavy-quark mass m_Q is taken to infinity. Then new symmetries of the strong interactions arise, which relate the long-distance properties of many observables to a small number of reduced hadronic matrix elements [1]–[6]. A systematic expansion around the heavy-quark limit has been applied successfully to learn about the properties of heavy mesons and baryons, such as their spectroscopy and decays.

A convenient tool to study the implications of the heavy-quark limit and to perform the $1/m_Q$ expansion is provided by the Heavy-Quark Effective Theory (HQET) [7], which is constructed by introducing a velocity-dependent field $h_v(x)$ related to the original heavy-quark field $Q(x)$ by

$$h_v(x) = e^{im_Q v \cdot x} \frac{1 + \not{v}}{2} Q(x), \quad (1)$$

so that $\not{v} h_v = h_v$. Here v is the 4-velocity of the hadron containing the heavy quark. The phase redefinition in (1) removes the large “mechanical” part $m_Q v$ of the heavy-quark momentum, which is due to the motion of the heavy hadron. The field $h_v(x)$ carries the “residual momentum” $k = p_Q - m_Q v$, which arises from the predominantly soft interactions of the heavy quark with gluons. The effective Lagrangian of the HQET is [8]–[10]

$$\mathcal{L}_{\text{eff}} = \bar{h}_v i v \cdot D h_v + \frac{1}{2m_Q} \bar{h}_v (iD_\perp)^2 h_v + C_{\text{mag}}(m_Q/\mu) \frac{g_s}{4m_Q} \bar{h}_v \sigma_{\mu\nu} G^{\mu\nu} h_v + O(1/m_Q^2), \quad (2)$$

where $D^\mu = \partial^\mu - ig_s A^\mu$ is the gauge-covariant derivative, and $D_\perp^\mu = D^\mu - (v \cdot D) v^\mu$ contains its components orthogonal to the velocity. The gluon field-strength tensor is defined as $[D^\mu, D^\nu] = -ig_s G^{\mu\nu}$. The origin of the operators arising at order $1/m_Q$ in (2) is most transparent in the rest frame of the heavy hadron: the first operator corresponds to the kinetic energy resulting from the residual motion of the heavy quark inside the hadron (note that $(iD_\perp)^2 = -(i\mathbf{D})^2$ in the rest frame), whereas the second operator describes the magnetic interaction of the heavy-quark spin with the gluon field. The Wilson coefficient C_{mag} results from short-distance effects and depends logarithmically on the heavy-quark mass and on the subtraction scale μ , at which the chromo-magnetic operator is renormalized [9]. As a consequence of a reparametrization invariance of the HQET, the kinetic operator is not renormalized [11].

In many phenomenological applications of the HQET, the forward matrix elements of the dimension-5 operators in (2) play a most significant role. They appear, for instance, in the spectroscopy of heavy hadrons [12]–[14], in the description of inclusive decay rates and spectra [15]–[21], as well as in the normalization of transition form factors at zero recoil [13]. For the ground-state pseudoscalar

and vector mesons, $M = P$ and V , one defines two hadronic parameters, λ_1 and $\lambda_2(\mu)$, by¹

$$\begin{aligned}\langle M(v) | \bar{h}_v (iD_\perp)^2 h_v | M(v) \rangle &= \lambda_1, \\ \langle M(v) | \bar{h}_v g_s \sigma_{\mu\nu} G^{\mu\nu} h_v | M(v) \rangle &= 2d_M \lambda_2(\mu),\end{aligned}\tag{3}$$

where we use a mass-independent normalization of states such that $\langle M(v) | \bar{h}_v h_v | M(v) \rangle = 1$. The coefficient d_M takes the values $d_P = 3$ and $d_V = -1$ for pseudoscalar and vector mesons, respectively. The μ dependence of the parameter λ_2 cancels against the μ dependence of the coefficient C_{mag} in (2). The product $C_{\text{mag}}\lambda_2$ is renormalization-group invariant. This quantity can be extracted from spectroscopy using the relation [13]

$$\frac{1}{4}(m_{B^*}^2 - m_B^2) = C_{\text{mag}}(1) \lambda_2(m_b) + O(\Lambda^3/m_b) = 0.12 \text{ GeV}^2.\tag{4}$$

In leading logarithmic approximation, one finds [9]

$$C_{\text{mag}}(m_Q/\mu) = \left(\frac{\alpha_s(m_Q)}{\alpha_s(\mu)} \right)^{3/\beta_0},\tag{5}$$

where $\beta_0 = 11 - \frac{2}{3}n_f$ is the first coefficient of the β function. Using this result, we obtain

$$\lambda_2(m_b) \simeq 0.12 \text{ GeV}^2, \quad \lambda_2(\mu_0) \simeq 0.15 \text{ GeV}^2,\tag{6}$$

where we have used the values $\alpha_s(m_b) = 0.21$ and $\alpha_s(\mu_0) = 0.4$, corresponding to a low renormalization point $\mu_0 \approx 1 \text{ GeV}$.

Although spectroscopic relations may be used to extract the differences of the matrix elements of the kinetic operator between different hadron states, the value of the parameter λ_1 itself cannot be determined from spectroscopy. Indeed, at present it is not even known whether λ_1 is a “physical” parameter in the sense that it can be defined unambiguously in a non-perturbative way. It may be that any such definition is intrinsically ambiguous because of the presence of infrared renormalons; however, it may also happen that for some fortuitous reason renormalons are absent in the case of the kinetic operator [22, 23]. From the practitioner’s point of view, λ_1 becomes a useful parameter once a scheme for treating perturbative corrections in the HQET has been specified. Here, we shall work in the $\overline{\text{MS}}$ subtraction scheme and assume that our value of λ_1 is used in connection with theoretical expressions that have one-loop perturbative accuracy.

Theoretical information about the parameter λ_1 can be obtained from a non-perturbative evaluation of the first matrix element in (3). Using QCD sum rules in the HQET, Ball and Braun have derived the value $-\lambda_1 = (0.52 \pm 0.12) \text{ GeV}^2$ [24], which has been adopted subsequently in many phenomenological analyses.

¹Another common notation is to define $\mu_\pi^2 = -\lambda_1$ and $\mu_G^2 = 3\lambda_2$.

This value is surprisingly large; it implies an average momentum of the heavy quark inside the meson of order 600–800 MeV. In fact, an earlier QCD sum-rule calculation using a less sophisticated approach had given the smaller value $-\lambda_1 = (0.18 \pm 0.06) \text{ GeV}^2$ [25]. A theoretical argument in favour of a smaller value of the kinetic energy was presented in Ref. [26]: the field-theory analog of the virial theorem relates the first matrix element in (3) to a matrix element of an operator containing the gluon field-strength tensor, making explicit an “intrinsic smallness” of λ_1 . On the other hand, a large value of $-\lambda_1$ was argued for by Voloshin and by Bigi et al., who derived the lower bound $-\lambda_1 > 3\lambda_2 \simeq 0.36 \text{ GeV}^2$ using first a quantum-mechanical reasoning [27] and later field-theoretical arguments based on zero-recoil sum rules [28]. Recently, however, it was shown that this bound is weakened significantly by higher-order perturbative corrections [29]. Thus, small values of $-\lambda_1$ can no longer be excluded a priori. Several authors have attempted to extract λ_1 (together with the “binding energy” $\bar{\Lambda}$) from a combined analysis of inclusive decay rates and moments of the decay spectra in beauty and charm decays [30]–[33]. The most recent of such analyses give values $-\lambda_1 \approx 0.1\text{--}0.2 \text{ GeV}^2$. A value of λ_1 has also been extracted using a lattice simulation of the HQET, yielding $-\lambda_1 = (-0.09 \pm 0.14) \text{ GeV}^2$ [34].

In view of these developments, it seems worth while to reconsider the problem of calculating λ_1 and λ_2 using QCD sum rules. Here we present the results of a new analysis, which is based on the virial theorem [13, 26].² Our approach has the advantage that both parameters are determined simultaneously from the zero-recoil (i.e. equal-velocity) limit of the QCD sum rule for the matrix element of a local dimension-5 operator between two meson states moving at different velocities. As a result, λ_1 and λ_2 are obtained from a set of two sum rules with very similar systematic uncertainties.

2 Derivation of the Sum Rules

The central object of our study is the meson matrix element of a local dimension-5 operator containing two heavy-quark fields at different velocities. Using the covariant tensor formalism of the HQET [36], we write

$$\langle M'(v') | \bar{h}_{v'} \Gamma i g_s G^{\mu\nu} h_v | M(v) \rangle = -\text{Tr} \left\{ \phi^{\mu\nu}(v, v') \overline{\mathcal{M}}'(v') \Gamma \mathcal{M}(v) \right\}, \quad (7)$$

where Γ is an arbitrary Dirac matrix, and

$$\mathcal{M}(v) = \frac{1 + \not{v}}{2\sqrt{2}} \begin{cases} \gamma_5; & \text{pseudoscalar meson } P(v) \\ \not{\epsilon}; & \text{vector meson } V(v, \epsilon) \end{cases} \quad (8)$$

is a matrix representing the spin wave-function of a ground-state meson M moving at velocity v . The most general decomposition of the tensor form factor $\phi^{\mu\nu}$

²Preliminary results of this analysis have been presented in Ref. [35].

consistent with Lorentz covariance and heavy-quark symmetry reads

$$\phi^{\mu\nu}(v, v') = \phi_1(w) (v^\mu v'^\nu - v^\nu v'^\mu) + \phi_2(w) [(v - v')^\mu \gamma^\nu - (v - v')^\nu \gamma^\mu] + \phi_3(w) i\sigma^{\mu\nu}, \quad (9)$$

where $w = v \cdot v'$. For equal velocities ($w = 1$) only the last term appears, and comparing with (3) we obtain the normalization condition

$$\phi_3(1) = \lambda_2. \quad (10)$$

The equations of motion of the HQET imply another normalization condition for a certain combination of the invariant functions $\phi_i(w)$. It reads [13]

$$3\phi_1(1) - 3\phi_2(1) - \frac{3}{2}\phi_3(1) = -\lambda_1. \quad (11)$$

This relation is remarkable in that it relates the matrix element of the kinetic operator in (3) to a matrix element of the gluon field-strength tensor, in accordance with the picture that the residual motion of the heavy quark inside the meson is caused by its interactions with gluons. Eq. (11) can be interpreted as the field-theory analog of the virial theorem, which relates the kinetic energy to a matrix element of the “electric” components of the gluon field [26].

We shall now derive the Laplace sum rules for the invariant functions $\phi_i(w)$. The analysis proceeds in complete analogy to that of the Isgur–Wise function. For a detailed discussion of the procedure and notations, the reader is referred to Ref. [37]. We consider, in the HQET, the 3-point correlation function of the local operator appearing in (7) with two interpolating currents for the ground-state heavy mesons:

$$\begin{aligned} & \int dx dy e^{i(k' \cdot y - k \cdot x)} \langle 0 | T \left\{ \bar{q} \bar{\Gamma}_{M'} h_{v'}(y), \bar{h}_v \Gamma i g_s G^{\mu\nu} h_v(0), \bar{h}_v \Gamma_M q(x) \right\} | 0 \rangle \\ &= \text{Tr} \left\{ \Phi^{\mu\nu}(v, v', k, k') \bar{\Gamma}_{M'} \frac{1 + \not{v}'}{2} \Gamma \frac{1 + \not{v}}{2} \Gamma_M \right\}, \end{aligned} \quad (12)$$

where k and k' are the external momenta. Depending on the choice $\Gamma_M = \gamma_5$ or $\Gamma_M = \gamma^\alpha - v^\alpha$, the heavy-light currents interpolate pseudoscalar or vector mesons, respectively. The Dirac structure of the correlator, as shown in the second line, is a consequence of the Feynman rules of the HQET. The quantity $\Phi^{\mu\nu}$ obeys a decomposition analogous to (9), with coefficient functions $\Phi_i(\omega, \omega', w)$ that are analytic in the “residual energies” $\omega = 2v \cdot k$ and $\omega' = 2v' \cdot k'$, with discontinuities for positive values of these variables. These functions also depend on the velocity transfer $w = v \cdot v'$.

The idea of QCD sum rules is to relate a theoretical approximation to the Operator Product Expansion (OPE) of the above correlator to a hadronic representation of the same correlator in terms of physical intermediate states. The lowest-lying states are the ground-state mesons $M(v)$ and $M'(v')$ associated with

the heavy-light currents. They lead to a double pole located at $\omega = \omega' = 2\bar{\Lambda}$, where $\bar{\Lambda} = m_M - m_Q$ is the “effective mass” of the ground-state mesons in the HQET [38]. The residue of this double pole is proportional to the invariant functions $\phi_i(w)$. We find

$$\Phi_i^{\text{pole}}(\omega, \omega', w) = \frac{\bar{\Lambda} \phi_i(w) F^2}{(\omega - 2\bar{\Lambda} + i\epsilon)(\omega' - 2\bar{\Lambda} + i\epsilon)}, \quad (13)$$

where F is the meson decay constant in the HQET ($F \simeq f_M \sqrt{m_M}$). In the deep Euclidean region the correlator can be calculated perturbatively because of asymptotic freedom. The main assumption behind QCD sum rules is that, at the transition from the perturbative to the non-perturbative regime, confinement effects can be described by including the leading power corrections in the OPE. They are proportional to vacuum expectation values of local quark–gluon operators, the so-called condensates [39]. Following the standard procedure, we write the theoretical expressions for Φ_i as double dispersion integrals and perform a Borel transformation in the variables ω and ω' . This eliminates possible subtraction polynomials and yields an exponential damping factor in the dispersion integrals, which suppresses the contributions from excited states. Because of heavy-quark symmetry, it is natural to set the associated Borel parameters equal: $\tau = \tau' \equiv 2T$. Following Refs. [37, 40], we then introduce new variables $\omega_+ = \frac{1}{2}(\omega + \omega')$ and $\omega_- = \omega - \omega'$, perform the integral over ω_- , and employ quark–hadron duality to equate the remaining integral over ω_+ up to a “continuum threshold” ω_c to the Borel transform of the double-pole contribution in (13). This yields the Laplace sum rules

$$\phi_i(w) F^2 e^{-2\bar{\Lambda}/T} = \int_0^{\omega_c} d\omega_+ e^{-\omega_+/T} \bar{\rho}_i(\omega_+, w). \quad (14)$$

The spectral densities $\bar{\rho}_i(\omega_+, w)$ arise after integration of the double spectral densities over ω_- .

As pointed out above, the theoretical expressions on the right-hand side of the sum rules consist of perturbative and non-perturbative contributions. The leading terms in the OPE arise from the diagrams shown in Fig. 1. In our analysis, we shall include the non-perturbative contributions of the quark condensate $\langle \bar{q}q \rangle$, the gluon condensate $\langle \alpha_s G^2 \rangle$, and the mixed quark–gluon condensate $\langle \bar{q} g_s \sigma_{\mu\nu} G^{\mu\nu} q \rangle \equiv m_0^2 \langle \bar{q}q \rangle$. For a consistent calculation at order α_s , we calculate the Wilson coefficients of the quark and gluon condensates to one-loop order, and the coefficient of the mixed condensate at tree level. At higher orders in the OPE, one encounters a proliferation of condensates whose values are essentially unknown. The terms of dimension six, in particular, consist of four-quark and three-gluon condensates. For an estimate of such contributions we include the effects of four-quark condensates, which arise from the diagram shown in

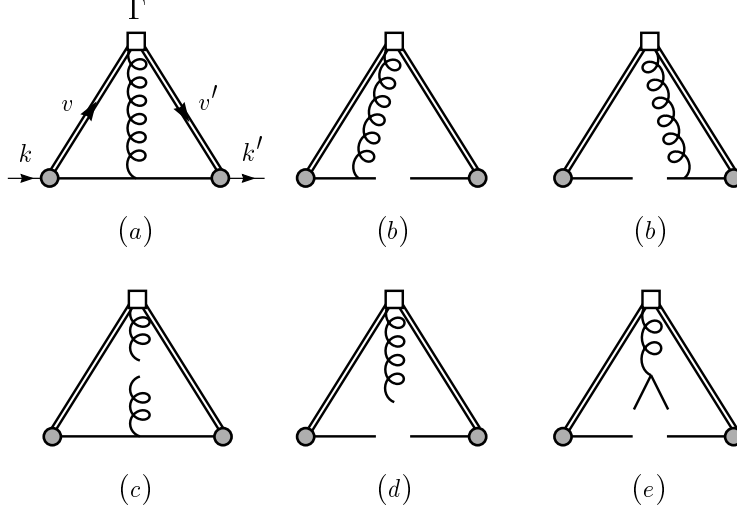


Figure 1: Non-vanishing diagrams for the 3-point correlator: (a) perturbative contribution, (b) quark-condensate, (c) gluon-condensate, (d) mixed-condensate, and (e) four-quark condensate contributions. The velocity-changing current operator is denoted by a white square, the interpolating meson currents by gray circles. Heavy-quark propagators are drawn as double lines.

Fig. 1(e). The calculation of the condensate terms is most conveniently performed using the fixed-point gauge $x \cdot A(x) = 0$ with the origin chosen at the position of the velocity-changing heavy-quark current. The most complicated part of the calculation is, however, to evaluate the perturbative contribution of the two-loop diagram shown in Fig. 1(a). We have calculated this diagram using the techniques developed in Ref. [41]. Our results for the Laplace sum rules are:

$$\begin{aligned}
\phi_1(w) F^2 e^{-2\bar{\Lambda}/T} &= \frac{2\alpha_s T^5}{\pi^3} \left(\frac{2}{w+1} \right)^2 \delta_4(\omega_c/T), \\
\phi_2(w) F^2 e^{-2\bar{\Lambda}/T} &= -\frac{2\alpha_s T^5}{\pi^3} \frac{2}{w+1} \delta_4(\omega_c/T) + \frac{4\alpha_s T^2}{3\pi} \langle \bar{q}q \rangle \delta_1(\omega_c/T) \\
&\quad - \frac{T}{48\pi} \frac{2}{w+1} \langle \alpha_s G^2 \rangle \delta_0(\omega_c/T) + \frac{\langle O_6 \rangle}{12T}, \\
\phi_3(w) F^2 e^{-2\bar{\Lambda}/T} &= \frac{4\alpha_s T^5}{\pi^3} \frac{2}{w+1} \delta_4(\omega_c/T) - \frac{8\alpha_s T^2}{3\pi} \langle \bar{q}q \rangle \delta_1(\omega_c/T) \\
&\quad + \frac{T}{24\pi} \frac{2}{w+1} \langle \alpha_s G^2 \rangle \delta_0(\omega_c/T) - \frac{m_0^2 \langle \bar{q}q \rangle}{12} - \frac{\langle O_6 \rangle}{12T}. \quad (15)
\end{aligned}$$

The functions $\delta_n(\omega_c/T)$ arise from the continuum subtraction and are given by

$$\delta_n(x) = \frac{1}{n!} \int_0^x dt t^n e^{-t} = 1 - e^{-x} \sum_{k=0}^n \frac{x^k}{k!}. \quad (16)$$

The four-quark condensate $\langle Q_6 \rangle$ is defined as

$$\langle Q_6 \rangle = g_s^2 \langle \bar{q} \gamma^\mu t_a q \sum_f \bar{f} \gamma_\mu t_a f \rangle \equiv -\frac{16\pi}{9} \kappa \alpha_s \langle \bar{q} q \rangle^2. \quad (17)$$

Assuming factorization of the four-quark operator [39] corresponds to setting $\kappa = 1$.

In the next step, we evaluate the sum rules in (15) for $w = 1$ and use the normalization conditions (10) and (11) to obtain the Laplace sum rules for the hadronic parameters λ_1 and λ_2 . This leads to

$$\begin{aligned} -\lambda_1 F^2 e^{-2\bar{\Lambda}/T} &= \frac{6\alpha_s T^5}{\pi^3} \delta_4(\omega_c/T) + \frac{m_0^2 \langle \bar{q} q \rangle}{8} (1 - \varepsilon_6), \\ \lambda_2 F^2 e^{-2\bar{\Lambda}/T} &= \frac{4\alpha_s T^5}{\pi^3} \delta_4(\omega_c/T) - \frac{8\alpha_s T^2}{3\pi} \langle \bar{q} q \rangle \delta_1(\omega_c/T) \\ &\quad + \frac{T}{24\pi} \langle \alpha_s G^2 \rangle \delta_0(\omega_c/T) - \frac{m_0^2 \langle \bar{q} q \rangle}{12} (1 + \varepsilon_6), \end{aligned} \quad (18)$$

where

$$\varepsilon_6 = -\frac{16\pi}{9} \kappa \frac{\alpha_s \langle \bar{q} q \rangle}{m_0^2 T}. \quad (19)$$

For all reasonable values of the parameters, ε_6 is of order a few per cent, which is much less than the uncertainty in the parameter m_0^2 . Therefore, the contribution of the four-quark condensate can be safely neglected in the numerical analysis, and we shall set $\varepsilon_6 = 0$ hereafter. The sum rule for λ_2 (without the contribution of the four-quark condensate) coincides with the result derived by Ball and Braun [24]; our sum rule for λ_1 is new. Notice that the sum rule for λ_1 does not receive contributions from the quark and gluon condensates. This is a consequence of the fact that (in the fixed-point gauge) the light quark interacts only with the magnetic components of the gluon field [42], whereas the combination of form factors defining λ_1 in (11) corresponds to a matrix element of the electric components [26].

For the evaluation of the sum rules, it is convenient to eliminate the explicit dependence on the parameters F and $\bar{\Lambda}$ by using the well-known sum rule for the correlator of two heavy-light currents [43, 37, 44]:³

$$F^2 e^{-2\bar{\Lambda}/T} = \frac{3T^3}{4\pi^2} \delta_2(\omega_c/T) - \langle \bar{q} q \rangle + \frac{m_0^2 \langle \bar{q} q \rangle}{4T^2}. \quad (20)$$

Dividing the sum rules in (18) by the sum rule in (20), we obtain expressions for the parameters λ_1 and λ_2 as functions of the Borel parameter T and the continuum threshold ω_c . This procedure reduces the systematic uncertainties in

³Since the leading terms in (15) are proportional to α_s and we have no control over the $O(\alpha_s^2)$ corrections, we do not include the known $O(\alpha_s)$ corrections in (20) for consistency.

the calculation. Moreover, it eliminates the dependence on the parameter $\bar{\Lambda}$, which is known to suffer from a renormalon ambiguity problem [22, 45]. For the QCD parameters entering the theoretical expressions, we take the standard values [39]

$$\begin{aligned}\langle\bar{q}q\rangle &= -(0.23 \pm 0.02)^3 \text{ GeV}^3, \\ \langle\alpha_s GG\rangle &= (0.04 \pm 0.02) \text{ GeV}^4, \\ m_0^2 &= (0.8 \pm 0.2) \text{ GeV}^2,\end{aligned}\tag{21}$$

as well as $\alpha_s = 0.4$. These values refer to a renormalization scale $\mu_0 \simeq 2\bar{\Lambda} \approx 1$ GeV, which is appropriate for evaluating QCD sum rules in the HQET. We shall comment below on the sensitivity of our result to the choice of the condensate parameters.

The sum-rule parameters ω_c and T should, in principle, be determined in a self-consistent way by requiring optimal stability of the results under variations of the Borel parameter inside the region where the theoretical calculations are reliable. For too small values of T , the OPE diverges, whereas for large values of T the contributions to the sum rules from higher resonance states become more and more important. Unfortunately, the continuum-contamination problem is rather severe in the case of sum rules for the matrix elements of higher-dimensional operators such as λ_1 and λ_2 . This is exemplified by the leading perturbative contribution to the correlator in (12), which is proportional to

$$\frac{1}{4! T^5} \int_0^\infty d\omega_+ \omega_+^4 e^{-\omega_+/T} = \delta_4(\omega_c/T) + [1 - \delta_4(\omega_c/T)].\tag{22}$$

The first term on the right-hand side is assigned to the ground-state, whereas the second term is removed in the continuum subtraction. For the central values $T = 0.9$ GeV and $\omega_c = 2.0$ GeV determined below, the ground-state contribution is only 7.5% of the total perturbative contribution. For comparison, in the case of the sum rule in (20), the leading perturbative contribution to the correlator is proportional to $\delta_2(\omega_c/T) + [1 - \delta_2(\omega_c/T)]$, and the ground-state contribution amounts to 38%. For this reason, it is better to determine the allowed regions for the parameters ω_c and T by requiring stability of the sum rule (20) for the meson decay constant, and then to use the same values in the evaluation of the sum rules for λ_1 and λ_2 [24]. One finds that $\omega_c = (2.0 \pm 0.3)$ GeV, and the “stability window” for the Borel parameter is $0.6 \text{ GeV} < T < 1.2 \text{ GeV}$ [37, 44].

Using these ranges of parameters, together with the central values of the condensates given in (21), we obtain the results shown in Fig. 2. The sum rule for the parameter describing the chromo-magnetic interaction of the heavy quark exhibits very good stability. Taking an average over the sum-rule window, we obtain

$$\lambda_2(\mu_0) = (0.19 \pm 0.02 \pm 0.02) \text{ GeV}^2,\tag{23}$$

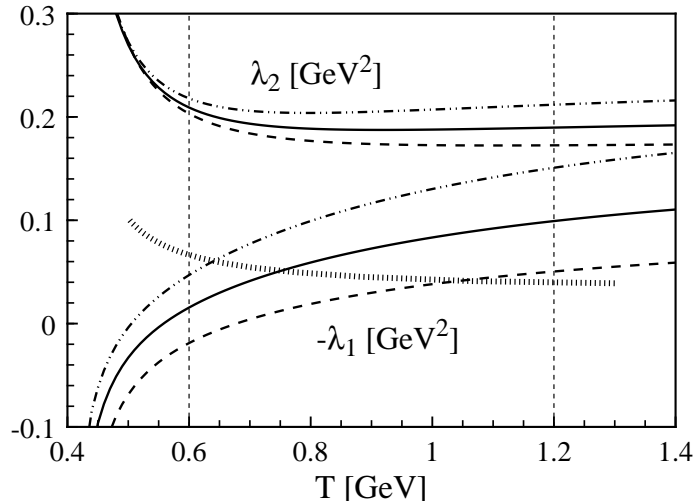


Figure 2: Sum-rule results for the parameters $-\lambda_1$ (lower curves) and λ_2 (upper curves). For each quantity, the three curves correspond to the following values of the continuum threshold: $\omega_c = 1.7$ GeV (dashed), 2.0 GeV (solid), 2.3 GeV (dash-dotted). The vertical dashed lines show the sum-rule window.

where the first error reflects the variation with ω_c and T , while the second error takes into account the uncertainty in the values of the vacuum condensates. When evolved to a high renormalization point, our result corresponds to $\lambda_2(m_b) = (0.15 \pm 0.03)$ GeV², which is in good agreement with the value in (6) extracted from spectroscopy. A very similar result has been obtained by Ball and Braun using the same approach [24], and by the present author using a different analysis based on two-point sum rules [46].

The stability of the sum rule for the kinetic-energy parameter λ_1 is not quite as good. The reason is that the condensate contribution has the opposite sign of the perturbative contribution. Inside the allowed parameter space for ω_c and T , we find values for $-\lambda_1$ ranging from -0.02 GeV² to $+0.15$ GeV². The dependence on the Borel parameter is strongest in the region of low T values, where the contribution of the mixed condensate becomes very large. If we restrict ourselves to the region of larger T values by requiring that the condensate term be less than 50% of the perturbative contribution, we find that the region below the hatched line in Fig. 2 is excluded. This leads to

$$-\lambda_1 \approx (0.10 \pm 0.05 \pm 0.02) \text{ GeV}^2, \quad (24)$$

where the second error reflects again the dependence on the condensate parameters. It must be stressed that because of the relatively poor stability the sum-rule prediction for λ_1 is affected by systematic uncertainties that may be underestimated by the error quoted in (24). Keeping this reservation (which applies equally to previous sum-rule determinations of λ_1) in mind, we note that our value in

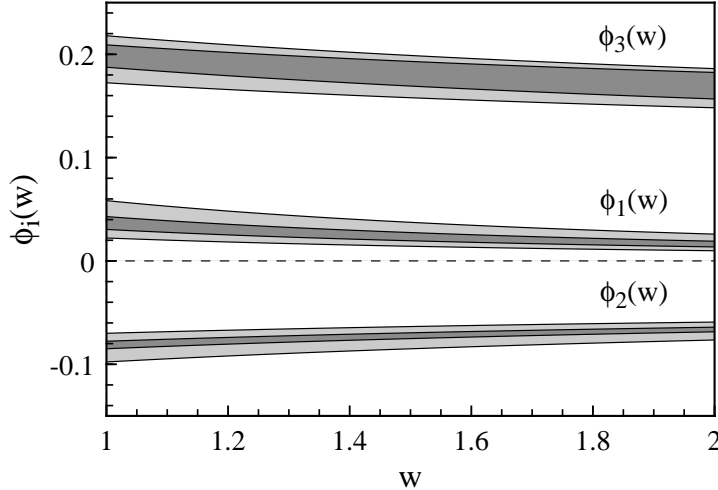


Figure 3: Sum-rule results for the functions $\phi_i(w)$. The width of the bands reflects the variation of the results with the continuum threshold ($1.7 \text{ GeV} < \omega_c < 2.3 \text{ GeV}$ for the light bands, and $\omega_c = 2 \text{ GeV}$ for the inner, dark bands) and the Borel parameter ($0.6 \text{ GeV} < T < 1.2 \text{ GeV}$).

(24) implies an average momentum of the heavy quark inside the meson of order 200–400 MeV, which appears to us to be a reasonable value. Clearly, our result is much smaller than the value $-\lambda_1 = (0.52 \pm 0.12) \text{ GeV}^2$ obtained by Ball and Braun [24]; indeed, we find $-\lambda_1 < \lambda_2(\mu_0)$ for all choices of the parameters. We shall comment below on the difference between their approach and ours.

Although our main focus was to derive sum rules for the parameters λ_1 and λ_2 , the invariant form factors $\phi_i(w)$ defined in (7) may be of some interest as well. For instance, the combination $f(w) = 3\phi_3(w) - 2(w-1)\phi_2(w)$ appears in the analysis of non-factorizable contributions to class-I non-leptonic two-body decays such as $\bar{B}^0 \rightarrow D^+\pi^-$ [47]. Therefore, we find it worth while to study the w dependence of these form factors using the sum rules in (15) combined with the sum rule in (20). In Fig. 3, we show the results for the functions $\phi_i(w)$ obtained by varying the parameters ω_c and T in the ranges described above. For all three functions, we observe a mild decrease with w . We note that these results can be trusted only for moderate values of w , which is however sufficient for all practical purposes. For $w \gg 1$, the non-perturbative contributions to the sum rules would rapidly vanish once the non-locality of the condensates was taken into account.

3 Concluding Remarks

We have presented a simultaneous determination of the HQET parameters λ_1 and λ_2 from a QCD sum-rule analysis of a 3-point correlation function containing the

local operator $\bar{h}_v \Gamma i g_s G^{\mu\nu} h_v$ together with two heavy-light currents. Our sum rule for the parameter λ_2 , which describes the chromo-magnetic interaction of a heavy quark, agrees with a result derived previously in Ref. [24]. However, our sum rule for λ_1 is new. It incorporates the virial theorem, which relates the kinetic energy of a heavy quark inside a meson to its chromo-electric interaction with gluons [26]. We shall argue below that our approach is superior to that of Ball and Braun [24], who extracted λ_1 from the correlator of the kinetic operator $\bar{h}_v (iD_\perp)^2 h_v$ with two heavy-light currents. The reason is that the virial theorem makes explicit an “intrinsic smallness” of λ_1 , which is otherwise hidden by a large background from excited-state contributions. Our numerical results are given in (23) and (24). The value of λ_1 implies an average residual momentum of the heavy quark inside a meson of order 200–400 MeV; the result for λ_2 translates into a spin splitting of $m_{B^*}^2 - m_B^2 = (0.60 \pm 0.12) \text{ GeV}^2$, which is in good agreement with experiment.

We like to add a final comment regarding the difference between our approach and that of Ball and Braun [24], who obtained a much larger value for λ_1 than our result in (24). The reason is that their sum rule contains a large contribution from a “bare” quark loop, which is $O(\alpha_s^0)$. Before the continuum subtraction, their result reads⁴

$$-\lambda_1 F^2 e^{-2\bar{\Lambda}/T} + C(T) = \frac{9T^5}{4\pi^2} - \frac{3}{8} m_0^2 \langle \bar{q}q \rangle + O(\alpha_s), \quad (25)$$

where $C(T)$ represents the contributions of excited states, which are removed in the continuum subtraction. We have argued in Ref. [26] that the virial theorem, which relates λ_1 to a matrix element of the gluon field-strength tensor, does not allow terms not containing the gauge coupling in the sum rule for λ_1 . We shall now explain how this statement is consistent with (25).

To start with, let us stress that we do not claim that the authors of Ref. [24] made a calculational mistake; indeed, we have checked that their result (25) is correct. What we are going to argue is that the leading perturbative term on the right-hand side of (25), together with part of the contribution from the mixed condensate, must not be attributed to the ground-state, but rather to excited states. The virial theorem helps to avoid from the start any subtleties related to the complicated problem of the continuum subtraction in 3-point sum rules.

In order to explain our argument, it is necessary to go into some of the details of QCD sum-rule calculations in the HQET. The sum rules for the matrix elements of local dimension-5 operators are closely related to the derivatives of the sum rules for some lower-dimensional operators with respect to the Borel parameters. Consider the 3-point sum rules for the meson decay constant (i.e. the sum rule for the Isgur–Wise function evaluated at zero recoil) and for the

⁴For simplicity, we do not display the terms of $O(\alpha_s)$ here, since the main problem is to understand the origin of the leading term.

product $\xi_-(1)F^2$, where $\xi_-(1)$ is the zero-recoil limit of a form factor defined in terms of the matrix element of the operator $\bar{h}_v \Gamma i D^\mu h_v$ [38]. These sum rules read [37]

$$\begin{aligned} F^2 e^{-2\bar{\Lambda}(t+t')} + C_1(t+t') &= \frac{3}{4\pi^2} \frac{1}{(t+t')^3} - \langle \bar{q}q \rangle + \frac{m_0^2 \langle \bar{q}q \rangle}{4} (t+t')^2 + O(\alpha_s), \\ \xi_-(1) F^2 e^{-2\bar{\Lambda}(t+t')} + C_2(t, t') &= \frac{3}{16\pi^2} \frac{7t' - t}{(t+t')^5} - \frac{m_0^2 \langle \bar{q}q \rangle}{24} (7t' - t) + O(\alpha_s), \end{aligned} \quad (26)$$

where $t = 1/\tau$ and $t' = 1/\tau'$ are the two Borel parameters associated with the variables ω and ω' , and C_i denote the contributions to the correlators from excited states. As a consequence of the orthogonality of states, $C_1(t+t')$ is a function of the sum of the Borel parameters only. Note, however, that the second sum rule is not symmetric in the two Borel variables. This is a consequence of the fact that the operator whose matrix element defines $\xi_-(w)$ contains a derivative acting on one of the heavy-quark fields. Consequently, the function $C_2(t, t')$ is not symmetric in its arguments. Taking derivatives with respect to the Borel parameters t and t' , we can derive a set of related sum rules containing powers of the parameter $\bar{\Lambda}$. In the case of the first sum rule in (26), it clearly does not matter whether we take a derivative with respect to t or t' ; however, the same statement is not true in the case the second sum rule. After taking the derivatives, we set the the Borel parameters equal, $t = t' = 1/2T$, in which case the first sum rule in (26) reduces to (20). We obtain

$$\begin{aligned} F^2 e^{-2\bar{\Lambda}/T} + C_1(1/T) &= \frac{3T^3}{4\pi^2} - \langle \bar{q}q \rangle + \frac{m_0^2 \langle \bar{q}q \rangle}{4T^2} + O(\alpha_s), \\ \bar{\Lambda} F^2 e^{-2\bar{\Lambda}/T} - \frac{1}{2} C_1'(1/T) &= \frac{9T^4}{8\pi^2} - \frac{m_0^2 \langle \bar{q}q \rangle}{4T} + O(\alpha_s), \\ \bar{\Lambda}^2 F^2 e^{-2\bar{\Lambda}/T} + \frac{1}{4} C_1''(1/T) &= \frac{9T^5}{4\pi^2} + \frac{m_0^2 \langle \bar{q}q \rangle}{8} + O(\alpha_s), \end{aligned} \quad (27)$$

and

$$\begin{aligned} \xi_-(1) F^2 e^{-2\bar{\Lambda}/T} + C_2(1/2T, 1/2T) &= \frac{9T^4}{16\pi^2} - \frac{m_0^2 \langle \bar{q}q \rangle}{8T} + O(\alpha_s), \\ \bar{\Lambda} \xi_-(1) F^2 e^{-2\bar{\Lambda}/T} - \frac{1}{2} \partial_t C_2(1/2T, 1/2T) &= \frac{3T^5}{2\pi^2} - \frac{m_0^2 \langle \bar{q}q \rangle}{48} + O(\alpha_s), \\ \bar{\Lambda} \xi_-(1) F^2 e^{-2\bar{\Lambda}/T} - \frac{1}{2} \partial_{t'} C_2(1/2T, 1/2T) &= \frac{3T^5}{4\pi^2} + \frac{7}{48} m_0^2 \langle \bar{q}q \rangle + O(\alpha_s). \end{aligned} \quad (28)$$

It is crucial that the continuum contribution $C_2(t, t')$ is not symmetric in t and t' ; otherwise the last two sum rules would be inconsistent with each other.

Using the equations of motion of the HQET, one can show that $\xi_-(1) = \bar{\Lambda}/2$ [38]. In fact, this relation follows by comparing the first sum rule in (28) with the second sum rule in (27), provided we identify $C_2(1/2T, 1/2T) = -\frac{1}{4} C_1'(1/T)$.

This relation between the continuum contributions is indeed satisfied when one adopts the standard continuum model, according to which

$$C_1(1/T) = \frac{3T^3}{4\pi^2} [1 - \delta_2(\omega_c/T)] , \quad C_2(1/2T, 1/2T) = \frac{9T^4}{16\pi^2} [1 - \delta_3(\omega_c/T)] . \quad (29)$$

Things are more subtle for the last two sum rules in (28), which have the same ground-state contribution. Inserting the normalization condition $\xi_-(1) = \bar{\Lambda}/2$, we find that their sum agrees with the last sum rule in (27). However, the only logical explanation for the fact that the theoretical expressions on the right-hand sides of these sum rules do not coincide is that the difference between these expressions contributes to the excited states only, but not to the ground state. Hence, taking the difference between the two sum rules in (28) leads to a sum rule with vanishing ground-state contribution. It reads

$$(\partial_{t'} - \partial_t) C_2(1/2T, 1/2T) = \frac{3T^5}{2\pi^2} - \frac{m_0^2 \langle \bar{q}q \rangle}{3} + O(\alpha_s) . \quad (30)$$

Let us now come back to the sum rule (25) for the matrix element of the kinetic operator derived by Ball and Braun. We can combine it with the above relation in such a way that the contribution of the bare quark loop, which is forbidden by the virial theorem, is eliminated from the right-hand side of the sum rule. Then the result takes the form

$$-\lambda_1 F^2 e^{-2\bar{\Lambda}/T} + \tilde{C}(T) = \frac{m_0^2 \langle \bar{q}q \rangle}{8} + O(\alpha_s) , \quad (31)$$

where the new continuum contribution is given by $\tilde{C}(T) = C(T) - \frac{3}{2}(\partial_{t'} - \partial_t) C_2(1/2T, 1/2T)$. What we have achieved is to identify the leading perturbative contribution in (25), as well as part of the contribution of the mixed condensate, as a contribution to the excited states coupling to the correlation function. What remains is nothing but our sum rule for λ_1 obtained in (18), with the correct coefficient in front of the mixed condensate.

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